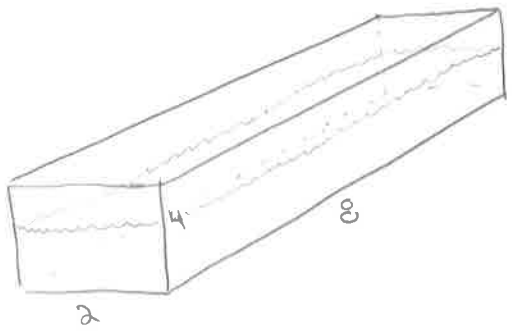
Calc AB: Related Rates · Supplementary Probs

① 

$V = l \cdot w \cdot h$

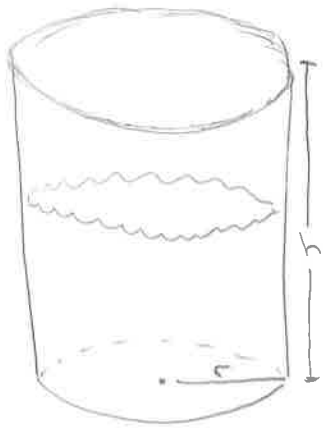
$\frac{dV}{dt} = l \cdot w \cdot \frac{dh}{dt}$

$2 = 8 \cdot 2 \cdot \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{1}{8}$ ft/min

know $\frac{dV}{dt}$, want $\frac{dh}{dt}$

when $h = 1$

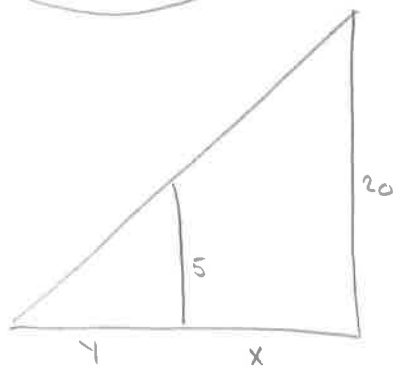
② 

$V = \pi r^2 h$ know $\frac{dV}{dt}$, want $\frac{dh}{dt}$ when $r = 6$

$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$

$2 = \pi r^2 \frac{dh}{dt}$ $\quad r = 6$

$\frac{dh}{dt} = \frac{2}{36\pi} = \frac{1}{9\pi}$ ft/min

③ 

know $\frac{dx}{dt} = 4$ want $\frac{dy}{dt}$

b)

$\frac{5}{y} = \frac{20}{x+y}$

$5x + 5y = 20y$

$5x = 15y$

$x = 3y$

$\frac{dx}{dt} = 3\frac{dy}{dt}$

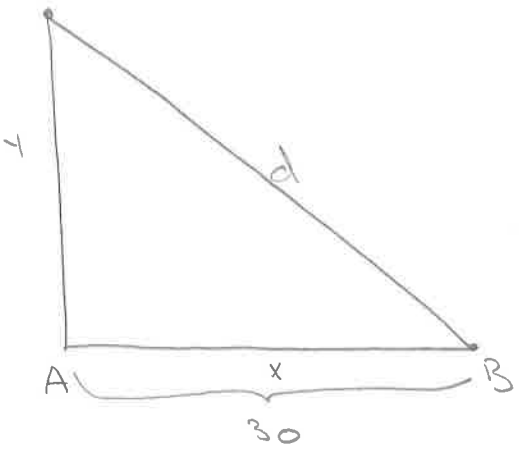
$4 = 3\frac{dy}{dt}$

$\frac{dy}{dt} = \frac{4}{3}$ ft/sec

a) tip moving

$\frac{d(x+y)}{dt} = \frac{dx}{dt} + \frac{dy}{dt} = 4 + \frac{4}{3} = \frac{16}{3}$ ft/sec

(4)



$\frac{dy}{dt} = 15$ ft/sec , $\frac{dx}{dt} = 0$

$x = 30$

$x^2 + y^2 = d^2$ $\quad y = 40$

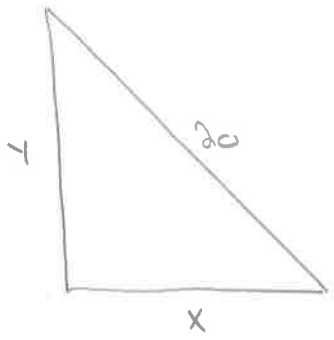
$d = 50$

$2x\frac{dx}{dx} + 2y\frac{dy}{dx} = 2d\frac{dd}{dx}$

$\frac{dd}{dx} = \frac{y \cdot \frac{dy}{dx}}{d} = \frac{40 \cdot 15}{50}$

$= 12$ ft/sec

(5)



a) $\frac{dx}{dt} = 2\frac{ft}{sec}$

$x = 12 \Rightarrow y = 16$

$x^2 + y^2 = 20^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{-x\frac{dy}{dx}}{y} = \frac{-12 \cdot 2}{16} = \frac{-24}{16} = -\frac{3}{2}$ ft/sec.

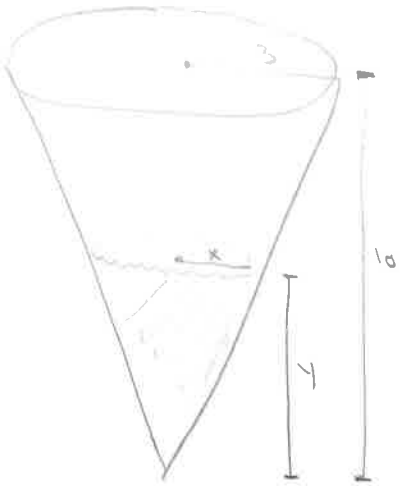
b) want $\frac{d\left(\frac{y}{x}\right)}{dt} = \frac{x\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}$

$= \frac{12\left(-\frac{3}{2}\right) - 16(2)}{144}$

$= \frac{-18 - 32}{144} = \frac{-50}{144}$

$= \frac{25}{72}$ ft/sec

(6)



$\frac{dV}{dt} = 4 \text{ ft}^3/\text{min}$

$V = \frac{1}{3}\pi x^2 y$

$\frac{x}{3} = \frac{y}{10}$

$10x = 3y$

$x = \frac{3}{10} y$

$V = \frac{1}{3}\pi \frac{9}{100} y^3$

$\frac{dV}{dt} = \frac{9}{100}\pi y^2 \frac{dy}{dt}$

$4 = \frac{9}{100}\pi y^2 \frac{dy}{dt}$

$\frac{400}{9\pi} = y^2 \frac{dy}{dt}$

$\frac{dy}{dt} = \frac{400}{9\pi \cdot 36}$

$\frac{dy}{dt} = \frac{100}{81\pi} \text{ ft/min} = .393 \text{ ft/min}$

b) want $\frac{dx}{dt}$

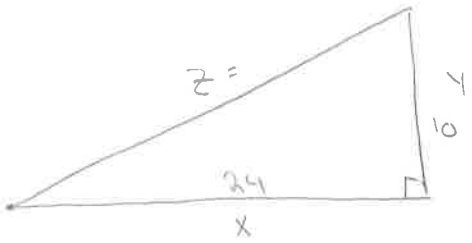
$x = \frac{3}{10} y$

$\frac{dx}{dt} = \frac{3}{10} \frac{dy}{dt}$

$= \frac{3}{10} \cdot \frac{100}{81\pi}$

$= \frac{10}{27\pi} \text{ ft/min}$

(7)



want $\frac{dz}{dt}$

when $x = 24$

$z = 26$

$\frac{dx}{dt} = \frac{3}{4}$ ft/sec

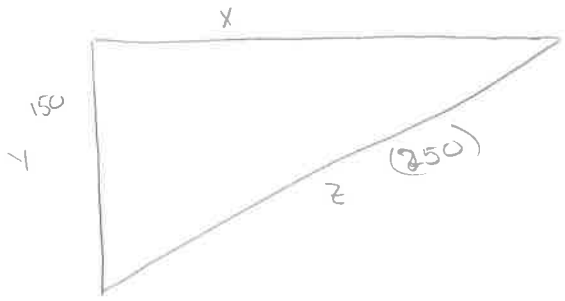
$x^2 + y^2 = z^2$

$x^2 + 100 = z^2$

$2x \frac{dx}{dt} + 0 = 2z \frac{dz}{dt}$

$$\frac{dz}{dt} = -\frac{x \frac{dx}{dt}}{z} = \frac{24 \cdot \frac{3}{4}}{26} = \frac{18}{26} = -\frac{9}{13} \text{ ft/sec}$$

(8)



$\frac{dx}{dt} = 20$ ft/sec want $\frac{dz}{dt}$

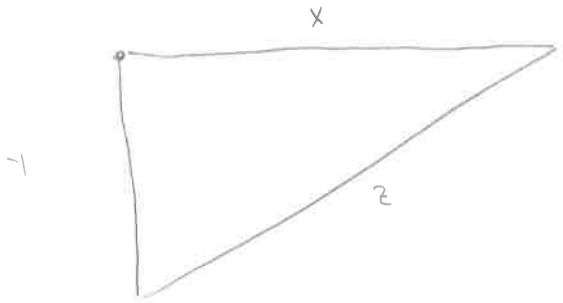
$x^2 + y^2 = z^2$

$x^2 + (150)^2 = z^2$ $\quad z = 250 \Rightarrow x = 200$

$2x \frac{dx}{dt} + 0 = 2z \frac{dz}{dt}$

$$\frac{dz}{dt} = -\frac{x \frac{dx}{dt}}{z} = \frac{200(20)}{250} = 16 \text{ ft/sec}$$

(9)



$\frac{dx}{dt} = 45$ $\quad \frac{dy}{dt} = 60$ $\quad$ want $\frac{dz}{dt}$

$x^2 + y^2 = z^2$

EB : $t = 4$

SB : $t = 3$

$x^2 + y^2 = z^2$

$180^2 + 180^2 = z^2$

$z = 180\sqrt{2}$

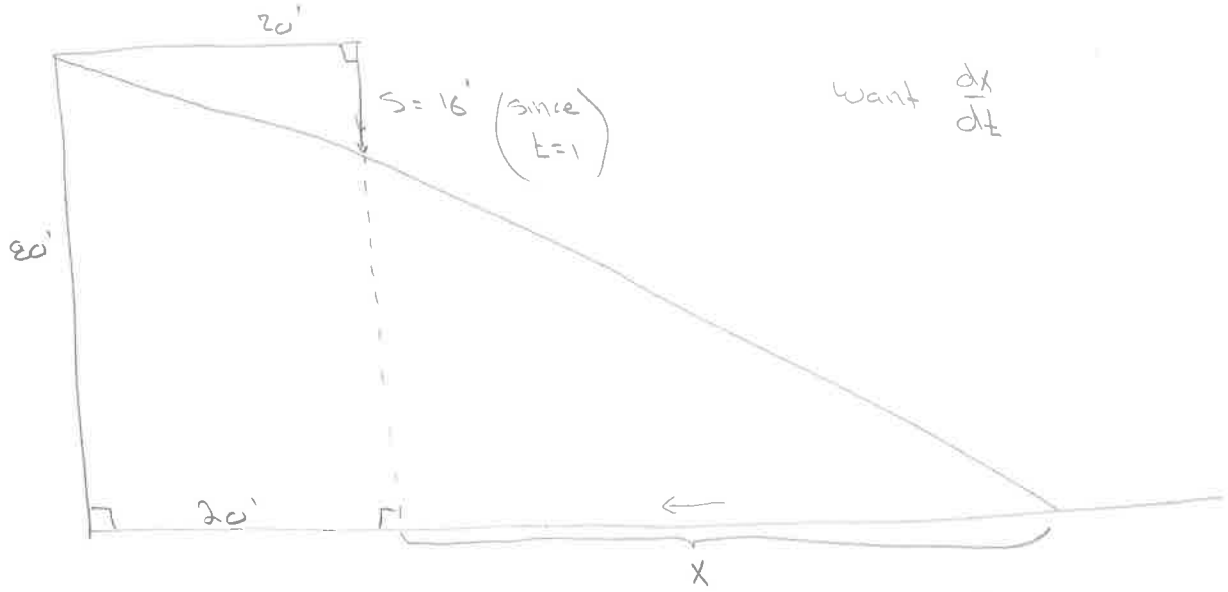
$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$$\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z} =$$

$$= \frac{180(45)' + 180(60)}{180\sqrt{2}}$$

$$= \frac{180(45+60)}{180\sqrt{2}} = \frac{105}{\sqrt{2}} = \frac{105\sqrt{2}}{2} \text{ mph}$$

(10)



want $\frac{dx}{dt}$

$\frac{s}{20} = \frac{80}{20+x}$

$20s + sx = 1600$

$20(16) + 16x = 1600$

$x = 80$

$20\frac{ds}{dt} + s\frac{dx}{dt} + x\frac{ds}{dt} = 0$

$20(32) + 16\frac{dx}{dt} + 80(32) = 0$

$16\frac{dx}{dt} = -32(20+80)$

$= -200$ ft/sec

solve for $\frac{dx}{dt}$ look @ small changes in s

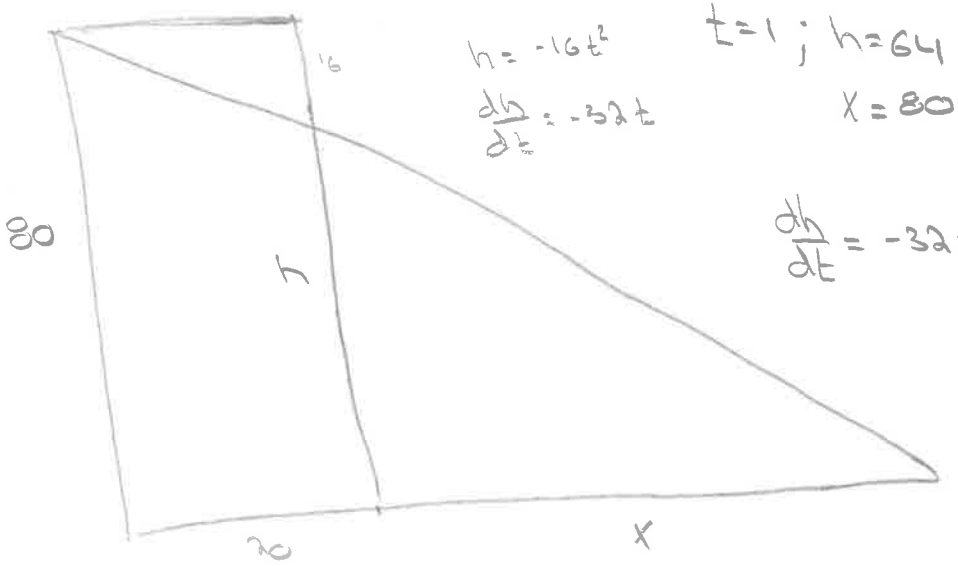
$sx = 1600 - 20s$

$x = 1600s^{-1} - 20$

$\frac{dx}{dt} = -\frac{1600}{s^2}\cdot\frac{ds}{dt}$

$= -200$

(15)



$h = -16t^2$

$\frac{dh}{dt} = -32t$

$t = 1$; $h = 64$

$x = 80$

$\frac{dh}{dt} = -32t$

$\frac{64}{x} = \frac{80}{20+x}$

$80x = 1280 + 64x$

$16x = 1280$

$x = 80$

$\frac{h}{x} = \frac{80}{20+x}$

$80x = 20h + hx$

$80\frac{dx}{dt} = 20\frac{dh}{dt} + h\frac{dx}{dt} + x\frac{dh}{dt}$

$\frac{dx}{dt} = \frac{20\frac{dh}{dt} + x\frac{dh}{dt}}{80 - h}$

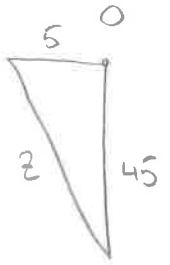
$\frac{dx}{dt} = \frac{\frac{dh}{dt}(20+x)}{80-h}$

$\left.\frac{dx}{dt}\right|_{t=1} = \frac{-32(20+80)}{80-64}$

$= \frac{-3200}{16}$

$= -200$ ft/sec

(11)



a) $x^2 + y^2 = z^2$ want $\frac{dz}{dt}$

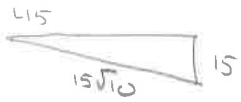
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z}$$

$$= \frac{5(-20) + 45(15)}{5\sqrt{82}}$$

$$= \frac{115}{\sqrt{82}} \text{ mph} \quad \text{approaching}$$

b) $$\frac{dz}{dt} = \frac{45(-20) + 15(15)}{15\sqrt{10}}$$



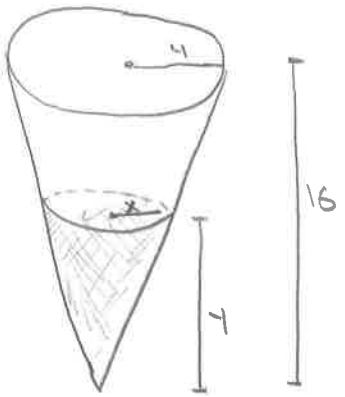
$$= \frac{15[3(-20) + 15]}{15\sqrt{10}}$$

$$= \frac{-60+15}{\sqrt{10}} = -\frac{45}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{-45\sqrt{10}}{10} = \frac{-9\sqrt{10}}{2} \text{ mph}$$

→ seperating

c) omit

(12)



$\frac{dV}{dt} = 10 \text{ ft}^3/\text{min}$

at $y = 12$, $\frac{dy}{dt} = 4 \text{ in/min} = \frac{1}{3} \text{ ft/min}$

$V = \frac{1}{3}\pi x^2 y$

$\frac{4}{x} = \frac{16}{y}$

$16x = 4y$

$x = \frac{1}{4}y$

$V = \frac{1}{3}\pi \frac{1}{16} y^3$

$V = V_{in} - V_{out}$

$V_{out} = V_{in} - V$

$\frac{dV_{out}}{dt} = 10 - \frac{dV}{dt}$

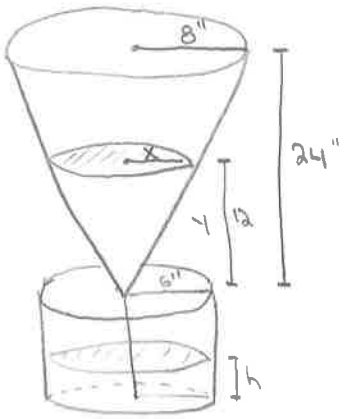
$\frac{dV}{dt} = \frac{1}{16}\pi y^2 \frac{dy}{dt}$

$\frac{dV}{dt} = \frac{1}{16} \cdot \pi (144) \frac{1}{3}$

$\frac{dV}{dt} = \frac{144}{48}\pi = 3\pi$

$\frac{dV_{out}}{dt} = 10 - 3\pi \text{ ft}^3/\text{min}$

(13)



Want $\frac{dh}{dt}$

$\frac{dy}{dt} = 1$

$\frac{8}{x} = \frac{24}{y}$

$24x = 8y$

$x = \frac{1}{3}y$

$V = \frac{1}{3}\pi x^2 y$

$= \frac{1}{3} \cdot \pi \frac{1}{9} y^3$

$= \frac{1}{27}\pi y^3$

$\frac{dV}{dt} = \frac{1}{9}\pi y^2 \frac{dy}{dt}$

$= \frac{1}{9}\pi \cdot 144 \cdot 1$

$= 16\pi$

$V = \pi r^2 h$

$V = 36\pi h$

$\frac{dV}{dt} = 36\pi \frac{dh}{dt}$

$16\pi = 36\pi \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{4}{9}$ in/min